EFFECT OF INELASTIC ELECTRON – ATOM COLLISIONS ON NONEQUILIBRIUM IONIZATION RATE

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We study possible formulations of the processes taking place near the cathode of the low-voltage arc as a function of the relationship between the electron Coulomb free path (l_{ee}) and the free path for elastic scattering of electrons by atoms (l_0) on the one hand, and for inelastic scattering (l_1), on the other hand. Expressions are obtained for the correction to the Maxwellian distribution function, the local and overall nonequilibrium ionization rate with atoms. Results of computer numerical calculations are presented.

In the overcompensated regime of a low-voltage arc, as a result of the cathode potential jump (E_0) the electron distribution function near the cathode is not Maxwellian. The magnitude of E_0 is less than the first atom-excitation energy level (E_1) and therefore direct ionization by electron impact is not possible. However the distribution function disturbances, localized initially for energies $E \sim E_0$, diffuse in energy space and partially in the region $E \geq E_1$. This leads to additional (nonequilibrium) ionization. With increasing distance from the cathode the distribution function disturbances relax, and ionization takes place as a result of the tail electrons of the Maxwellian distribution. As $E \to 0$ and $E \to \infty$ the electron-distribution function is Maxwellian at any distance from the cathode.

Following [1, 2], we can write the equations for the correction to the distribution function

$$\frac{\partial^2\psi}{\partial\xi^2} + \frac{\partial^2\psi}{\partial\eta^2} - \frac{\partial\psi}{\partial\eta} = 0 \quad \text{for} \quad E < E_1 \quad \left(\psi = \frac{f}{f_0} - 1\right) \tag{1}$$

and under the assumption that the nonelastic scattering cross section (σ_{01}) depends linearly on the electron energy near E_1

$$\frac{\partial^2\psi}{\partial\xi^2} + \frac{\partial^2\psi}{\partial\eta^2} - \frac{\partial\psi}{\partial\eta} - \gamma\left[\psi - (\nu - 1)\right] = 0 \quad \text{for} \quad E > E_1 \tag{2}$$

Here f is the true electron distribution function, f_0 is the Maxwellian function.

If removal of the ionization products takes place sufficiently rapidly, near the cathode $\psi \gg \gamma - 1$ and then in the region $E > E_1$

$$\frac{\partial^2\psi}{\partial\xi^2} + \frac{\partial^2\psi}{\partial\eta^2} - \frac{\partial\psi}{\partial\eta} - \gamma\psi = 0 \tag{3}$$

The boundary conditions at the cathode have the form [1]

$$\left.\frac{\partial\psi}{\partial\xi}\right|_{\xi=0} = 0 \quad \text{for} \quad E < E_0 \tag{4}$$

$$\left.\left(\psi - g\frac{\partial\psi}{\partial\xi}\right)\right|_{\xi=0} = B\tau^2 \exp\left(-(\tau - 1)(\eta - \eta_0)\right) - 1 \quad \text{for} \quad E > E_0 \tag{5}$$

$$\xi = x / L, \ \eta = E / T, \ \eta_0 = E_0 / T, \ \eta_1 = E_1 / T, \ \tau = T / T_c,$$

$$\gamma = \eta_1 l_{ee} / l_1 \quad g = 4L / l_{ee}, \ L = \sqrt{\eta_1 l_0 l_{ee} / 3}, \ l_0 = T_c / (p\sigma_0),$$

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$$l_1 = T_c / ((p\sigma_{01}(E_1 + T), \; l_{ee} = T^2 / (2\pi n e^4 \ln \Lambda)$$
$$B = J_0 / ((^1/_4)\, nv \exp(-\eta_0)), \; \sigma_0 = 3.5\cdot 10^{-14}\,\text{cm}^2, \sigma_{01} = 5\cdot 10^{-15}(E - E_1)\,\text{cm}^2$$

Here x is the distance to the cathode, T_C and T are respectively the cathode and plasma electron temperatures, n is the plasma density, p is the pressure of the neutral gas, J_0 is the Richardson current, v is the average plasma electron velocity, ν is the ratio of the true number of atoms in the first excited state to the thermodynamic equilibrium number, l_{ee} is the electron Coulomb mean-free path, l_1 and l_0 are respectively the mean-free paths for inelastic and elastic scattering of electrons by atoms.

Thus, the problem may be examined separately in the three regions $E < E_0$, $E_0 < E < E_1$, $E > E_1$ with corresponding differential equations and boundary conditions on ξ. The conditions for continuity of the function $\psi(\eta, \xi)$ and its first derivative with respect to η for $\eta = \eta_0$ and $\eta = \eta_1$ are imposed. The boundary conditions (4) and (5) require application of the Wiener–Hopf method and in this case the solution encounters serious difficulties. We can simplify the problem by examining two limiting cases:

(a) when $g \gg 1$, relatively low neutral gas pressures p, and high plasma density n;

(b) when $g \ll 1$, large pressure p, low density n.

As shown in [3], case (a) corresponds to low reflection coefficient of the electrons entering the plasma back to the cathode, case (b) corresponds to a high reflection coefficient. We note that the requirement $g \gg 1$ by virtue of $l_1 \gg l_0$ automatically leads to smallness of the parameter γ and vice versa. The experimental facts concerning the arc regime of the thermoemission converter [1, 4] indicate that the pressure p is sufficiently low (1-6 mm Hg) and the density n is sufficiently high (10^{13}-10^{14} cm^{-3}) that the condition $g \gg 1$ is satisfied. Thus, two formulations of the problem are possible:

(a) when $g \gg 1$, $\gamma < 1$ along with retention in (5) of the term with $\partial\psi / \partial\xi$ and neglect of ψ in comparison with this term;

(b) when $g \ll 1$, $\gamma > 1$ the derivative is dropped; case (a) is realized in the arc regime of the thermoemission converter.

In examining problem (a) there is no need to use the Wiener–Hopf method. Fourier transformation with respect to ξ and subsequent solution of the differential Eqs. (1) and (3) with corresponding boundary conditions yield the following expressions for the Fourier transforms of the corrections to the distribution function

$$\Phi_1(k, \varepsilon) = A_1(k)\exp(-p_0\varepsilon) + B_1(k)\exp(p_0\varepsilon) \quad \text{for} \quad E < E_0$$
$$\Phi_2(k, \varepsilon) = A_2(k)\exp(-p_0\varepsilon) + B_2(k)\exp(p_0\varepsilon) + \quad \text{for} \quad E_0 < E < E_1$$
$$+ \frac{\sqrt{2}}{(\sqrt{\pi}\, g)}\left[B\tau^2 \frac{\exp(-(\tau - ^1/_2)\,\varepsilon)}{p_0^2 - (\tau - ^1/_2)^2} - \frac{\exp(-\varepsilon/2)}{p_0^2 - ^1/_4}\right]$$
$$\Phi_3(k, \varepsilon_1) = A_3(k)\exp(-p_1\varepsilon_1) + B_3(k)\exp(p_1\varepsilon_1) \quad \text{for} \quad E > E_1$$
$$+ \frac{\sqrt{2}}{(\sqrt{\pi}\, g)}\left[B\tau^2 \frac{\exp(-(\tau - 1)\,\varepsilon_0 - (\tau - ^1/_2)\,\varepsilon_1}{p_1^2 - (\tau - ^1/_2)^2} - \frac{\exp(-\varepsilon_1/2)}{p_1^2 - ^1/_4}\right]$$

where the coefficients A_1, B_1, A_2, B_2, A_3, B_3 are determined from the condition of continuity of ψ and $\partial\psi/\partial\eta$ for $\eta = \eta_0$ and $\eta = \eta_1$ and from the condition $\psi \to 0$ as $E \to 0$ and $E \to \infty$

$$\varepsilon = \eta - \eta_0, \qquad \varepsilon_1 = \eta - \eta_1, \qquad \varepsilon_0 = \eta_1 - \eta_0$$
$$p_0 = \sqrt{k^2 + ^1/_4}, \quad p_1 = \sqrt{k^2 + \gamma + ^1/_4}$$

In the considered region $E > E_1$, after inverse transformation we obtain

$$\psi(\varepsilon_1, \xi, \tau) = \frac{1}{\pi g}\left[B\tau^2 I_1(\varepsilon_1, \xi, \tau) - I_1(\varepsilon_1, \xi, 1)\right]$$
$$I_1(\varepsilon_1, \xi, \tau) = \int_{-\infty}^{\infty}\left[\frac{\exp(-(\tau - 1)\,\varepsilon_0 - (p_1 - ^1/_2)\,\varepsilon_1 - ik\xi)}{(p_0 + p_1)(p - \tau + ^1/_2)}\right. \tag{6}$$
$$- \frac{\exp(-(p_0 - ^1/_2)\,\varepsilon_0 - (p_1 - ^1/_2)\,\varepsilon_1 - ik\xi)}{(p_0 + p_1)(p_0 - \tau + ^1/_2)} + \frac{\exp(-(\tau - 1)(\varepsilon_0 + \varepsilon_1) - ik\xi)}{p_1^2 - (\tau - ^1/_2)^2}$$
$$\left. - \frac{(p_0 + \tau - ^1/_2)\exp(-(\tau - 1)\,\varepsilon_0 - (p_1 - ^1/_2)\,\varepsilon_1 + ik\xi)}{(p_0 + p_1)(p_1^2 - (\tau - ^1/_2)^2)}\right] dk$$

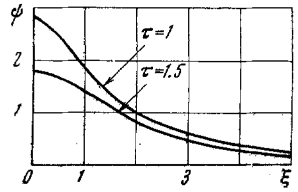

Fig. 1

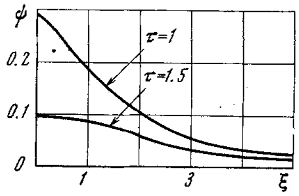

Fig. 2

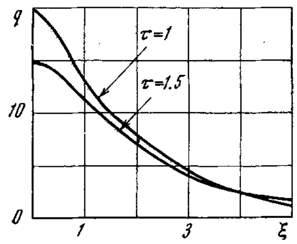

Fig. 3

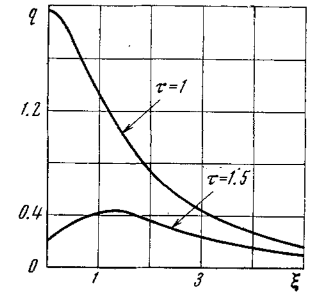

Fig. 4

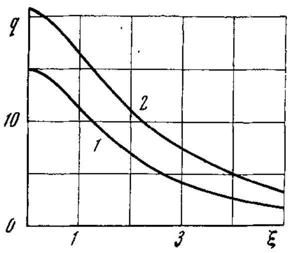

Fig. 5

Then the number of ions appearing per cm^3 per sec as a result of disturbance of the distribution function may be found by calculating the integral [1]

$$q(\xi, \tau) = \frac{nv \exp(-\eta_1)}{l_1} \int_0^\infty \exp(-\varepsilon_1)\, \psi(\varepsilon_1, \xi, \tau)(\varepsilon_1 + \eta_1)\varepsilon_1 d\varepsilon_1 \tag{7}$$

Integrating (7) over ξ, we obtain the following equation for the total number of ions appearing in the interelectrode space per cm^2 of cathode area per sec in the nonequilibrium ionization case

$$Q(\tau, \gamma) = \frac{nvL \exp(-\eta_1)}{gl_1} [B\tau^2 I_2(\tau, \gamma) - I_2(1, \gamma)] \tag{8}$$

where

$$I_2(\tau, \gamma) = (\eta_1/\beta^3 + 2/\beta^4)\left[\frac{1 - \exp(-(\tau-1)\varepsilon_0)}{\tau - 1} + \frac{\tau \exp(-(\tau-1)\varepsilon_0)}{\tau^2 - \tau - \gamma}\right] - (\eta_1/\tau^2 + 2/\tau^3)\frac{\exp(-(\tau-1)\varepsilon_0)}{\tau^2 - \tau - \gamma}$$

$$\beta = \sqrt{\gamma + {}^1/_4} + {}^1/_4$$

The constant B, written previously in terms of J_0, can be expressed in term of an easily measureable quantity – the arc current J

$$B = \frac{gJ}{({}^1/_3)\, nv\, (l_0/L)(\tau\eta_0 + 1)\exp(-\eta_0)} + \frac{\eta_0 + 1}{\tau\eta_0 + 1} \tag{9}$$

Now the expression for Q with account for inelastic electron–atom collisions has the form

$$Q(\tau, \gamma) = a(\tau, \gamma) J - b(\tau, \gamma) \tag{10}$$

where $a(\tau, \gamma)$ and $b(\tau, \gamma)$ are found from (8) and (9). It follows from (10) that for each choice of $\tau, \varepsilon_0, \gamma$ there is a minimal current J_* for which Q vanishes and then becomes negative (i.e., the total number of ions appearing in this case in the arc interelectrode space is less than in equilibrium). This effect is associated with leakage to the cathode of high-energy electrons as a consequence of $\tau > 1$ ($T > T_C$).

For $\tau = 1$, $\eta_0 + 1 \approx \eta_0$, $\eta_1 + 2 \approx \eta_1$ we can use in place of (10) the simpler expression

$$Q = J\frac{E_1}{E_0}\exp(-(E_1 - E_0)/T)\left[1 + \frac{\gamma}{\beta^3}\frac{E_1 - E_0}{T} - \frac{1}{\beta^3}\right] \tag{11}$$

Equation (11) reflects the basic characteristics of nonequilibrium ionization and for small γ agrees with the result obtained previously.

The calculations using (6) and (7) with account for (9) were made on a computer. The results are shown in Figs. 1-5. In the calculations we used the following values of the plasma parameters

$$n = 2 \cdot 10^{13} \text{cm}^{-3},\ p = 1.66 \text{ mm Hg}.$$
$$T = 2400° \text{ K},\ E_0 = 1 \text{ eV},\ E_1 = 1.39 \text{ eV}$$

Figures 1 and 2 show ψ versus the dimensionless coordinate $\xi = x/L$ for $\tau = 1$ (L = 0.0052 cm) for $\tau = 1.5$ (L = 0.0043 cm) with the energy $E = E_1 + T$ respectively for J = 2 A/cm^2 and J = 0.2 A/cm^2. Figures 3 and 4 show q verses ξ respectively for J = 2 A/cm^2 and J = 0.2 A/cm^2, where q is in C/cm^3 · sec. Figure 5 shows for comparison the curves $q(\xi)$ with (curve 1) and without (curve 2) account for inelastic electron–atom collisions (here J = 2 A/cm^2, $\tau = 1.5$)

In conclusion we shall present several estimates of $Q(\tau)$ using (10) for the previously indicated values of the plasma parameters for different J and τ

$$Q(1) = 0.215 \text{ C/cm}^2 \cdot \text{sec}$$
$$Q(1.5) = 0.145 \text{ C/cm}^2 \cdot \text{sec for } J = 2 \text{ A/cm}^2$$
$$Q(1) = 0.0215 \text{ C/cm}^2 \cdot \text{sec}$$
$$Q(1.5) = 0.0053 \text{ C/cm}^2 \cdot \text{sec for } J = 0.2 \text{ A/cm}^2$$

The limiting current J_* in the case $\tau = 1.5$ is 0.07 A/cm^2. However, if we consider inelastic collisions, then

$$Q(1) = 0.283 \text{ C/cm}^2 \cdot \text{sec}$$
$$Q(1.5) = 0.225 \text{ C/cm}^2 \cdot \text{sec for } J = 2 \text{ A/cm}^2$$
$$Q(1) = 0.0283 \text{ C/cm}^2 \cdot \text{sec}$$
$$Q(1.5) = 0.0092 \text{ C/cm}^2 \cdot \text{sec for } J = 0.2 \text{ A/cm}^2$$

Thus, the effect of inelastic collisions is significant and increases with increase of τ. This is explained by the fact that the length $l_1 \sim T_c$ and $l_{ee} \sim T^2$. Therefore

$$\gamma = \eta_1 l_{ee} / l_1 \sim T^2 / T_c$$

and the relative contribution of inelastic electron–atom collisions becomes greater with increase of τ in comparison with electron–electron collisions.

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